Optimization problems in energy economics: On the impact of tax uncertainty on optimal investment into carbon abatement technology

Rüdiger Frey ruediger.frey@wu.ac.at
joint work with Katia Colaneri, Tor Vergata and Verena Köck, WU

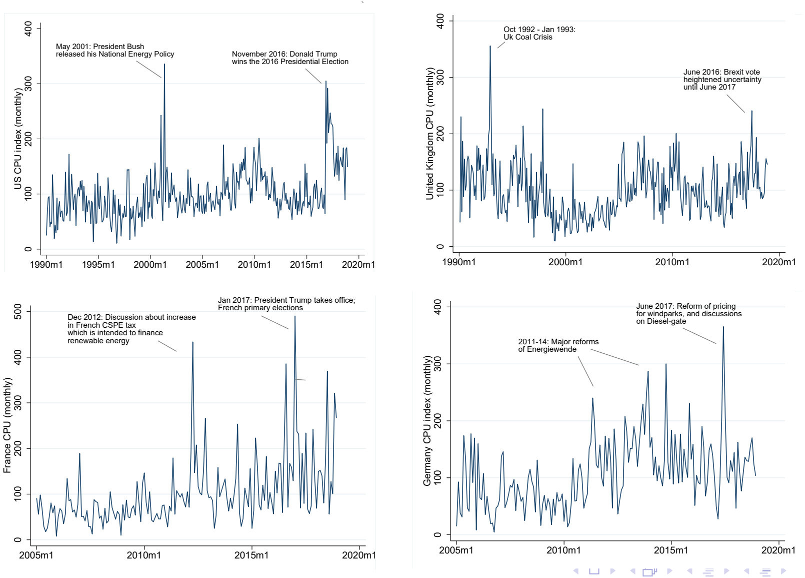
Vienna University of Economics and Business (WU)

QFin@Work, Tor Vergata, April 2023

Introduction

- According to many economists, carbon taxes and emissions trading (carbon price) are a key policy tool for fighting climate change (e.g. Nordhaus [1993], Golosov et al. [2014])
- Most of this work is concerned with optimal tax schemes for an efficient emission reduction (Nordhaus [1993], Golosov et al. [2014])
- In reality (environmental-) tax policy is affected by many uncertain factors such as political sentiment, outcome of elections, lobbying or international climate policy, so that future tax rates are random
- In fact Climate Policy Uncertainty and its impact on asset prices and investor decisions has become an active research topic and there are formal Climate Policy Uncertainty Indices

Climate Policy Uncertainty Index (Berestycki et al. (2022))



Our contribution

- We study how uncertainty about future carbon tax rates affect investment strategy of a stylized electricity producer who can invest in emission abatement technology
- Investments are done continuously in time but are irreversible and subject to transaction cost $\Rightarrow$ Producer is faced with a dynamic control problem.
- Mathematical contribution. Analysis of the ensuing control problem for jump diffusions (Characterization of value function, classical solutions etc.)
- Financial contribution (ongoing). Numerical experiments on the impact of various forms of tax- and price uncertainty and of the structure of production and abatement technology on investment into abatement technology.

Related work

- Fuss et al. [2008] Numerical analysis of the impact of policy uncertainty on investment in abatement technology in a real options model via discrete time dynamic programming; a related study by the International Energy Agency is Yang et al. [2008]
- Empirical studies on impact of carbon taxes include Aghion et al. [2016] and Martinsson et al. [2022].
- There is also an empirical literature on climate policy uncertainty and climate policy uncertainty indices

The model

- We consider a stylized monopolistic electricity producer, who has to decide on the amount $q \geq 0$ to be produced at every t and on investments into abatement technology.
- The producer pays taxes on emissions represented by tax rate τ.
- Instantaneous profit of the producer is given by the function

$$\Pi(q, I, \tau, y) = p(q, y)q - C(q, I, \tau, y) \tag{1}$$

  Here $p(q, y)$ is the inverse demand function and $C(q, I, \tau, y)$ the cost function for producing q units of electricity, given current investment level I and tax rate τ a
- At every t producer chooses q to maximise her instantaneous profit; optimal profit is

$$\Pi^*(I, \tau) = \max_{q \geq 0} \Pi(q, I, \tau). \tag{2}$$

- Often we consider the simpler case where p and q are fixed or where the factor process is not present.

Investment in abatement technology

- Producer chooses rate $\boldsymbol{\gamma} = (\gamma_t)_{t\geq 0}$ at which she invests in abatement technology. For a given strategy $\boldsymbol{\gamma}$, the investment I has dynamics

$$I_t = I_0 + \int_0^t \gamma_s \mathrm{d}s - \int_0^t \delta I_s \mathrm{d}s + \sigma W_t, \quad t \geq 0 \tag{3}$$

  where W is a Brownian motion, $0 \leq \delta < 1$ the depreciation rate and $\sigma \geq 0$ (typically small).
- We assume $\gamma_t \geq 0$ for all t (irreversible investment); $\mathcal{A}$ denotes the set of admissible strategies.
- Investment is subject to buildup- or transaction cost given by $\kappa\gamma^2$ (penalization of rapid build up of abatement technology).
- Investment is financed by borrowing at interest rate $r > 0$

Optimal investment problem

- Goal of the producer: choose strategy γ to maximize total profits up to time $T > 0$, that is

$$\max_{\gamma \in \mathcal{A}} \mathbb{E}_t \left[\int_t^T \left(\Pi^*(I_s, \tau_s) - \gamma_s - \kappa \gamma_s^2 \right) e^{-r(s-t)} \mathrm{d}s + e^{-r(T-t)} h(I_T) \right] \tag{4}$$

- $h(\cdot)$ accounts for the residual value of the abatement technology at time T.
- In the paper this problem is solved (numerically) via dynamic programming equation

Production function: filter technology

- Let X be the input, say, coal with price $\bar{c}$ per unit.
- Amount of emission (CO_2) per unit of X is e_0. Filters $\Rightarrow$ emissions are reduced by $e_1(I)$.
- Total emission: $e(X, I) = X(e_0 - e_1(I))$, where abatement function $e_1(\cdot)$ is increasing, concave and bounded by e_0
- $Q(X)$ is electricity that can be produced from X units coal, for $Q(\cdot)$ increasing and concave.
- This gives the following cost function for producing q units of electricity

$$C(q, I, \tau) = Q^{-1}(q)(\bar{c} + \tau(e_0 - e_1(I))), \tag{5}$$

Example 2: Two technologies

- The energy producer has access to two production technologies, e.g. coal or gas and solar panels.
- Gas costs $c_b(y)$ per unit and emits e_b tons of CO_2 per unit.
- $Q_b(X)$ electricity produced with X units of gas.
- Green production has zero marginal cost, does not emit CO_2.
- $c_g I$ electricity produced green for given investment I.
- Operating cost for green technology $C_0(I)$

$$C(q, I, \tau) = \begin{cases} C_0(I) & \text{if } q - c_g I \leq 0, \\ C_0(I) + (c_b(y) + e_b \tau) Q_b^{-1}(q - c_g I) & \text{if } \ q - c_g I > 0, \end{cases} \quad (6)$$

Tax rate

In the numerical experiments we consider scenarios with 2 states (values of the tax process) $\tau^1 = 0 < \tau^2$

- Random tax increase. Here $\tau_0 = 0$ but producer expects τ to increase to τ^2 at random later state, eg. as government implements international climate treaties; probability of upward jump in $(t, t+h]$ is approx. $g_{12}h$
- Tax reversal. Here τ is initially in the high-tax state τ^2, but energy producer expects a correction (jump to 0 at a later date) perhaps due to political lobbying or a change in government ("Trump after Biden"); probability of downward jump in $(t, t+h]$ is approx. $g_{21}h$

The tax scenarios

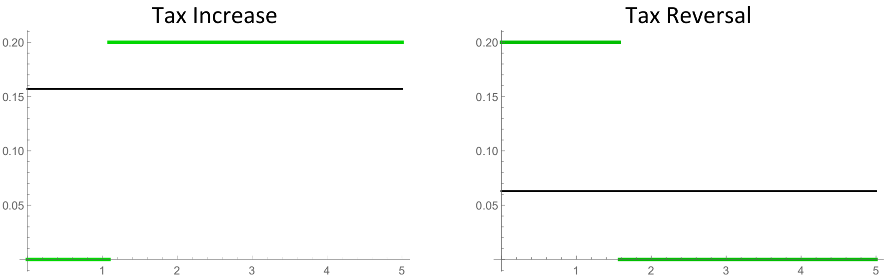


Figure: Tax policies. **black** deterministic tax rate, **green** random tax rate. In each panel the quantity $\mathbb{E}\left[\int_0^T \tau_s \mathrm{d}s\right]$ is identical for random and deterministic tax

The value function

- The value function is the optimal value (profit, utility etc) one can achieve if one starts at a given time and state.
- In our case

$$V(t, I, \tau) = \max_{\gamma \in \mathcal{A}} \mathbb{E}\Big[\int_t^T \left(\Pi^*(I_s, \tau_s) - \gamma_s - \kappa \gamma_s^2\right) e^{-r(s-t)} \mathrm{d}s \quad (7)$$

$$+ e^{-r(T-t)} h(I_T) \mid I_t = I, \tau_t = \tau \Big] \quad (8)$$

- In stochastic control one (tries to) characterize V by a partial differential equation, the HJB equation;
- Solving this PDE (usually numerically) gives information on the optimal value and the optimal strategy
- Famous example in finance: Merton portfolio problem of maximizing expected utility of a trading portfolio

The HJB equation

In our case the HJB equation is

$$v_t(t, I, \tau^0) + \Pi^*(I, \tau^1) - rv(t, I, \tau) + [v(t, I, \tau^2) - v(t, I, \tau^1)]g_{12}(t) \qquad (9)$$

$$+ \frac{\sigma^2}{2} v_{II}(t, I, \tau) + \sup_{0 \leq \gamma} v_I(t, I, \tau)(\gamma - \delta I) - (\gamma + \kappa\gamma^2) = 0 \qquad (10)$$

with the terminal condition $v(T, I, \tau) = h(I)$. Looks nice ...

Optimal strategy. The optimal investment rate is

$$\gamma^*(t, I, \tau, y) = (V_I(t, I, \tau) - 1)^+ / 2\kappa$$

(Trade-off between expected future profits and current cost.)

Numerical experiments: Setup and overview

- Throughout we consider the case where q is equal to $\bar{q} = 10$, $\delta = 0.1, \sigma = 0.05,\ T = 10$.
- Filter technology. *Cost function* is increasing and concave in I, residual value $h(I) = 0$;
- Two technologies: residual value $h(I) \approx I$.
- Tax rate: 2 states $\tau^1 = 0$, $\tau^2 > 0$, transition intensity $g_{12} = 0.25, g_{21} = 0$ (random tax increase) resp. $g_{21} = g_{12} = 0.25$

We show results on

- Optimal investment rate for different buildup cost κ
- Comparison of average investment and emission reduction to a deterministic scenario with same average tax rate for tax reversal and random tax increase scenario

Optimal investment for tax increase scenario (filter)

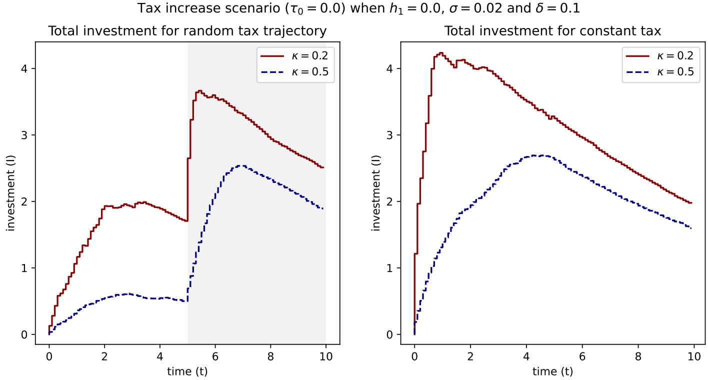


Figure: Optimal investment $I^*(t)$ for tax increase; left: random tax, right: constant tax. Note that there is a substantial amount of investment already before the jump in τ (hedging)

Optimal investment for tax reversal scenario (filter)

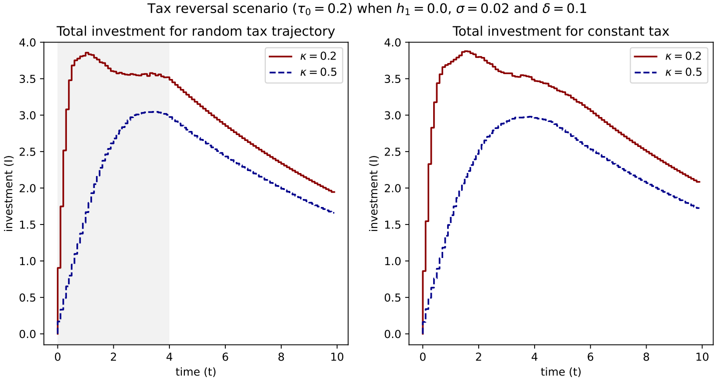


Figure: Optimal investment $I^*(t)$ for tax reversal; left: random tax, right: constant tax.

Optimal investment for both scenarios (2 technologies).

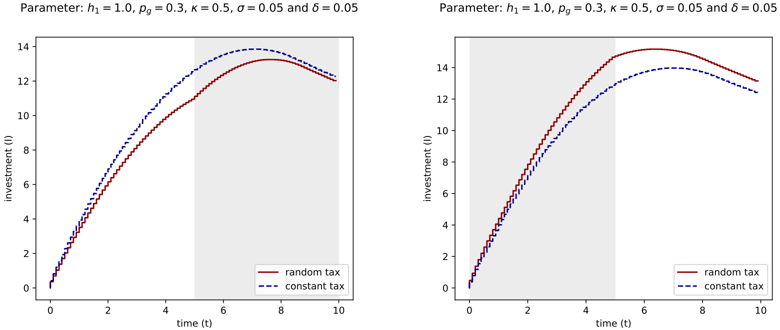


Figure: Optimal investment $I^*(t)$ for 2 technologies; left: tax increase, right: tax reversal The impact of taxes on γ resp. I is usually smaller than in filter case

Average emissions (filter)

κ	random	constant
0.2	5.45	3.75
0.5	8.90	6.76

κ	random	constant
0.2	4.25	3.83
0.5	7.20	6.07

Table: left: random tax increase; right: tax reversal. The constant tax leads on average to lower emissions in both cases.

For the two technology case there is no clear ordering of the different tax policies.

Summary and Conclusion

- For the filter technology random tax seems to be worse than deterministic benchmark;
- For 2 technologies on clearcut comparison possible; (investment mainly driven by low marginal cost of producing green)
- Results for the case with divisible investment (stochastic control) complement the real options approach of Fuss et al. [2008]. In particular, we see that investment buildup cost matter a lot.
- Further work (short term)
  - Study case with endogenous price and quantity
  - Exogenous noise in prices/demand function, possibly correlated with switching intensity
- Further work (longer term)
  - Cost of capital: higher interest rate for borrowing than for investing
  - Study case where investor learns switching intensity (filtering)
  - Equilibrium considerations (many small producers $\Rightarrow$ mean-field game)

P. Aghion, A. Dechezleprêtre, D. Hémous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1): 1–51, 2016. doi: 10.1086/684581.

Christian Beck, Sebastian Becker, Patrick Cheridito, Arnulf Jentzen, and Ariel Neufeld. Deep splitting method for parabolic PDEs. *SIAM Journal on Scientific Computing*, 43(5):A3135–A3154, 2021.

Rüdiger Frey and Verena Köck. Deep neural network algorithms for parabolic pides and applications in insurance and finance. *Computation*, 10, 2022. https://doi.org/10.3390/computation10110201.

S. Fuss, J. Szolgayova, and M. Obersteiner. Investment under market and climate policy uncertainty. *Applied Energy*, 85:708–721, 2008.

Maximilien Germain, Huyen Pham, and Xavier Warin. Approximation error analysis of some deep backward schemes for nonlinear pdes. *SIAM Journal of Scientific Computing*, 22:A28 –A56, 2022.

M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014.

O. A. Ladyzenskaja, V. A. Solonnikov, and N. N. Ural'ceva. *Linear and Quasilinear Equations of Parabolic Type*. American Mathematical Society, Providence, Rhode Island, 1968.

G. Martinsson, P. Stromberg, L. aszlo Sajtos, and C. Thomann. Carbon pricing and firm-level CO2 abatement: Evidence from a quarter of a century-long panel. working paper, European Corporate Governance Institute, 2022. Available at SSRN: https://ssrn.com/abstract=4206508 or http://dx.doi.org/10.2139/ssrn.4206508.

W. Nordhaus. Optimal greenhouse-gas reduction and tax policy in the 'DICE' model. *The American Economic Review*, 82:313–317, 1993.

Huyên Pham. Optimal stopping of controlled jump diffusion processes: a viscosity solution approach. *Journal of Mathematical Systems, Estimation and Control*, 8:1–27, 1998.

M. Yang, W. Blyth, and R. Bradley. Climate policy uncertainty and investment risk. Technical report, International Energy Association, Paris, 2008.