Behavioral risk modeling

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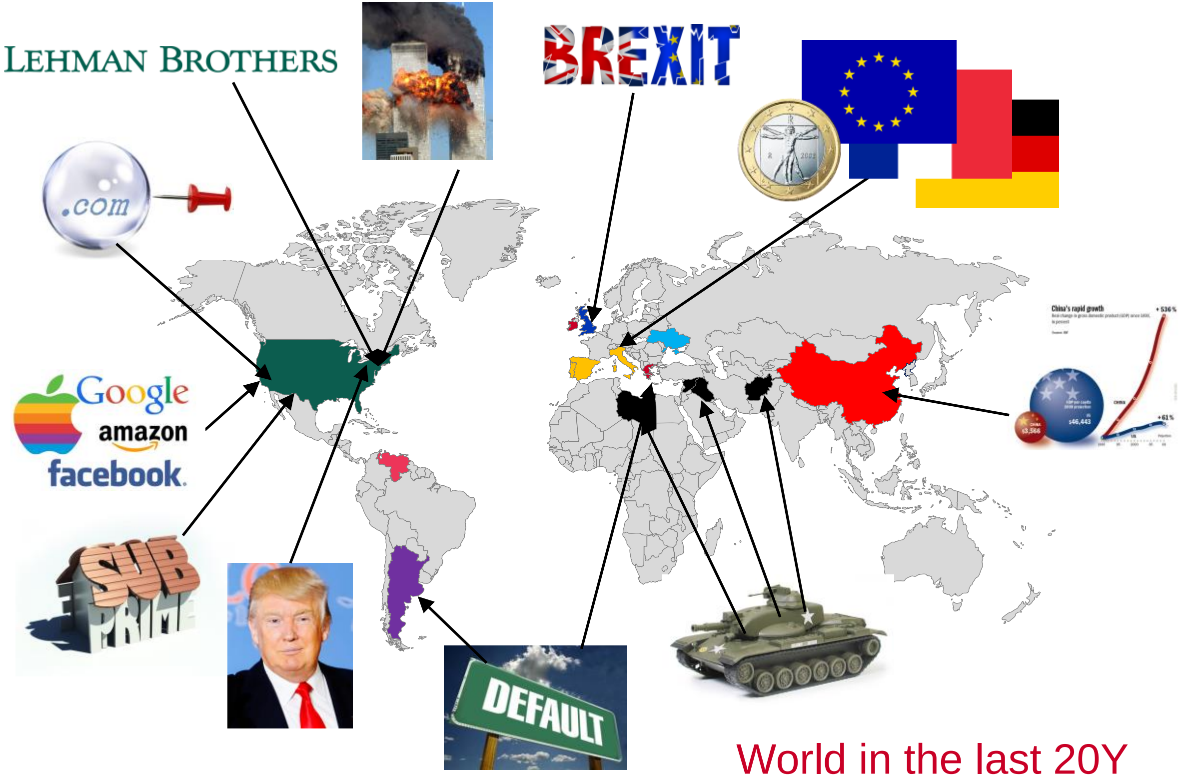
Modeling rational and irrational behavior

Foucault's pendulum

Dept. of Physics
Univ. of Rome La Sapienza
23/12/1996 – 23/12/2016



World in the last 20Y

Outline

1. A definition of behavioral risk
2. Overview of behavioral models
3. The calibration of risk premium and behavioral value adjustments
4. A mathematical framework in parallel with credit risk modelling
5. Behavioral risk and regulation (Basel IRRBB)

A. *Hints: A detailed example of hybrid intensity model (BIX)*

Behavioral risk in finance

Behavioral finance is a recent field in economics which studies the impact of psychology on the behavior of practitioners and financial markets from a general point of view

In a more specific contest, **behavioral risk** affects the valuation of financial instruments with **embedded options**, such as prepayment or extension options. A decision has to be taken, allowing one counterpart to terminate the contract or modify contractual conditions.

Behavioral risk arises whenever option holders do not act only on the strength of financial convenience, but follow an uncertain and sub-optimal exercise strategy if seen from the point of view of option seller

However, quite surprisingly, there is no unique definition:

- *Behavioral risk as **prepayment risk***
- *Behavioral risk as **residual risk***

Instruments with embedded options

Assets or liabilities with embedded prepayment/extension options subject to behavioral risk

Assets

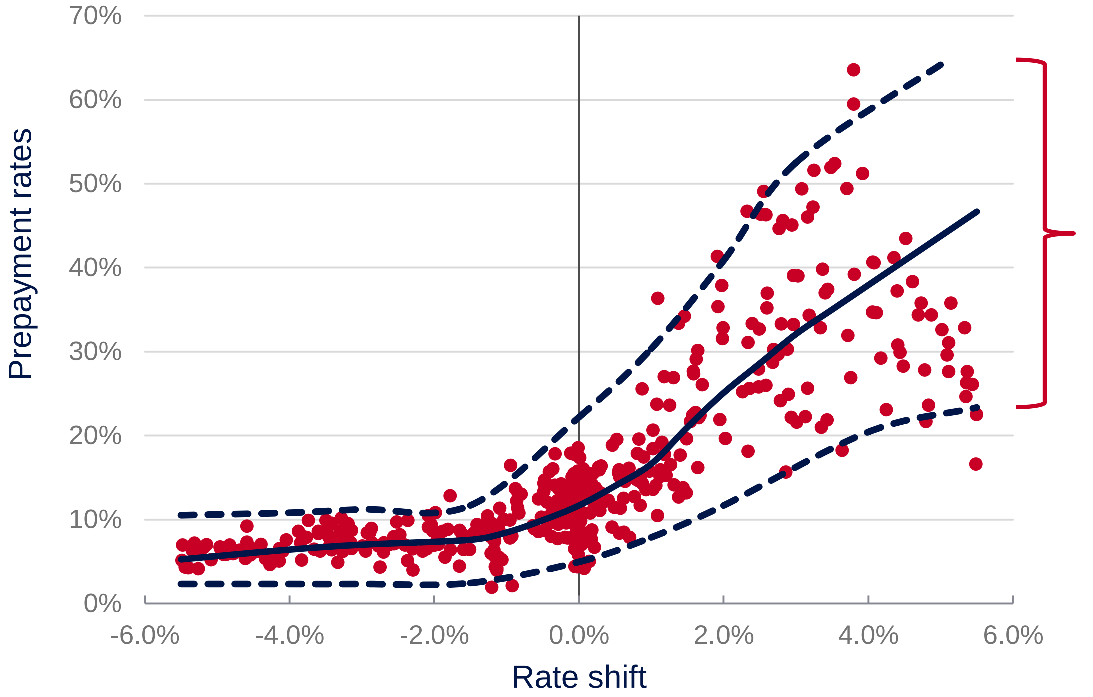
- *Mortgages, residential mortgages*
- *Mortgage-Backed Securities*
- *Callable bonds*
- *Corporate loans, retail loans*
- *Loan commitments*

Liabilities

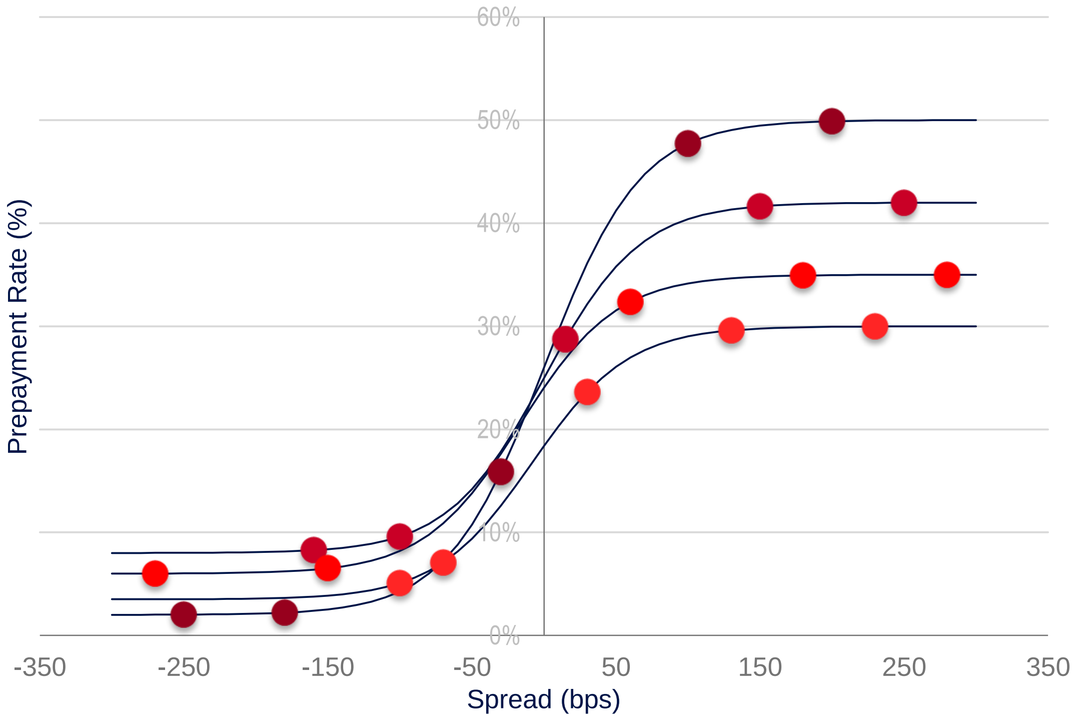
- *Short-sight/non-maturity deposits*
- *Puttable bonds*
- *Postal bonds (issued e.g. by CDP)*
- *Life insurance policies, annuities*

Empirical data of mortgage prepayments

- Some borrowers prepay when this is not convenient
- Some others do not prepay when this is convenient
- Prepayment rates exhibit **S-shaped dependence** on financial convenience measured by the rate shift
- **Residual variance** observed for the same market scenario
- Burnout effect as a function of loan age



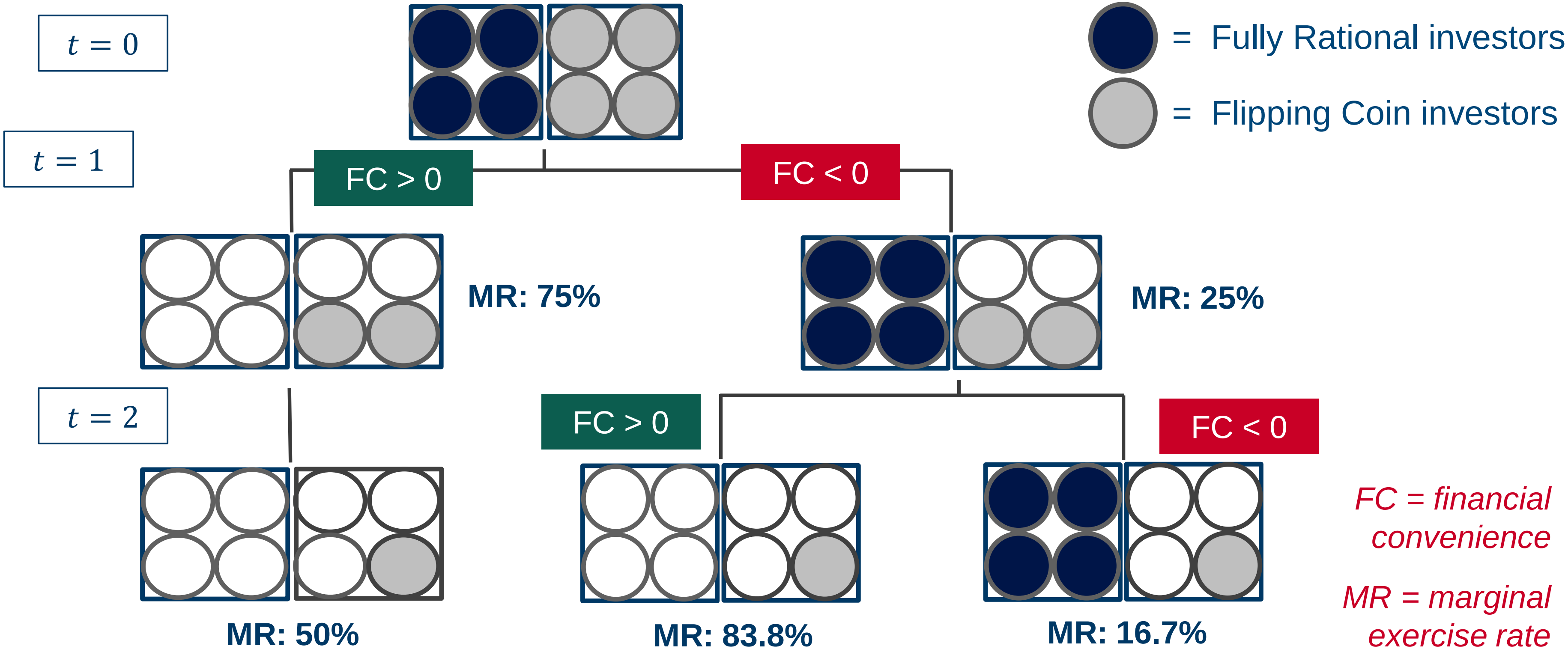
Heterogeneity



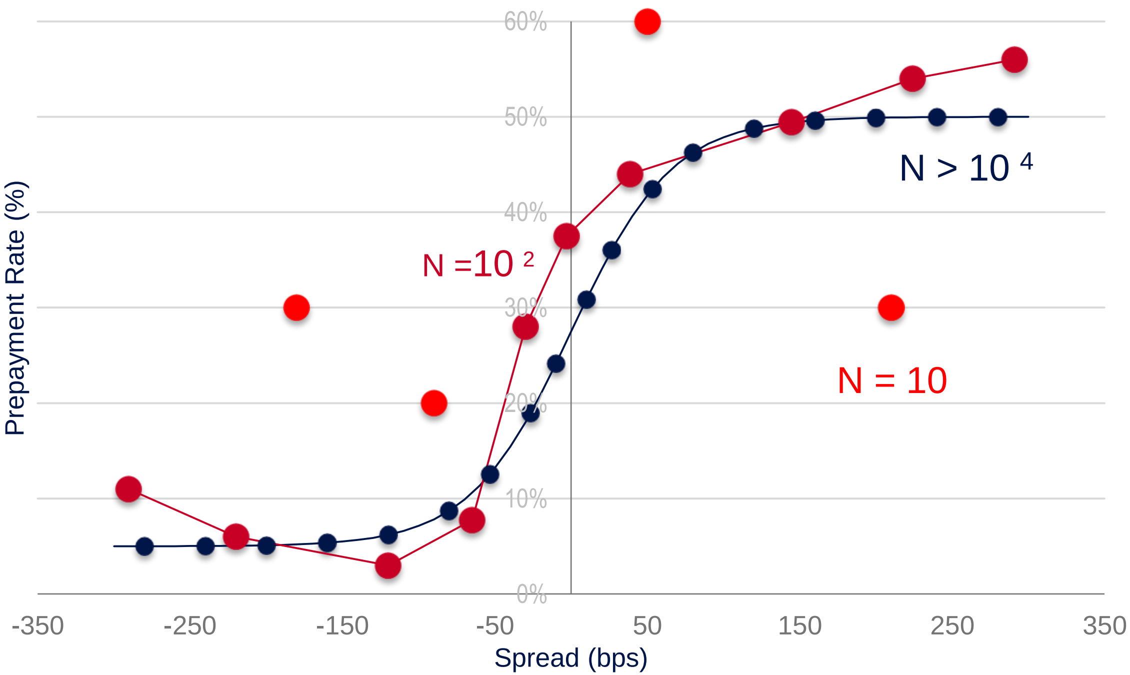
Heterogenous behavior of option holders within different pools of mortgages:

- More or less rational borrowers
- Different responsiveness to changing market conditions
- Different transaction costs
- Dependence on loan age

Time dependency due to burnout effect



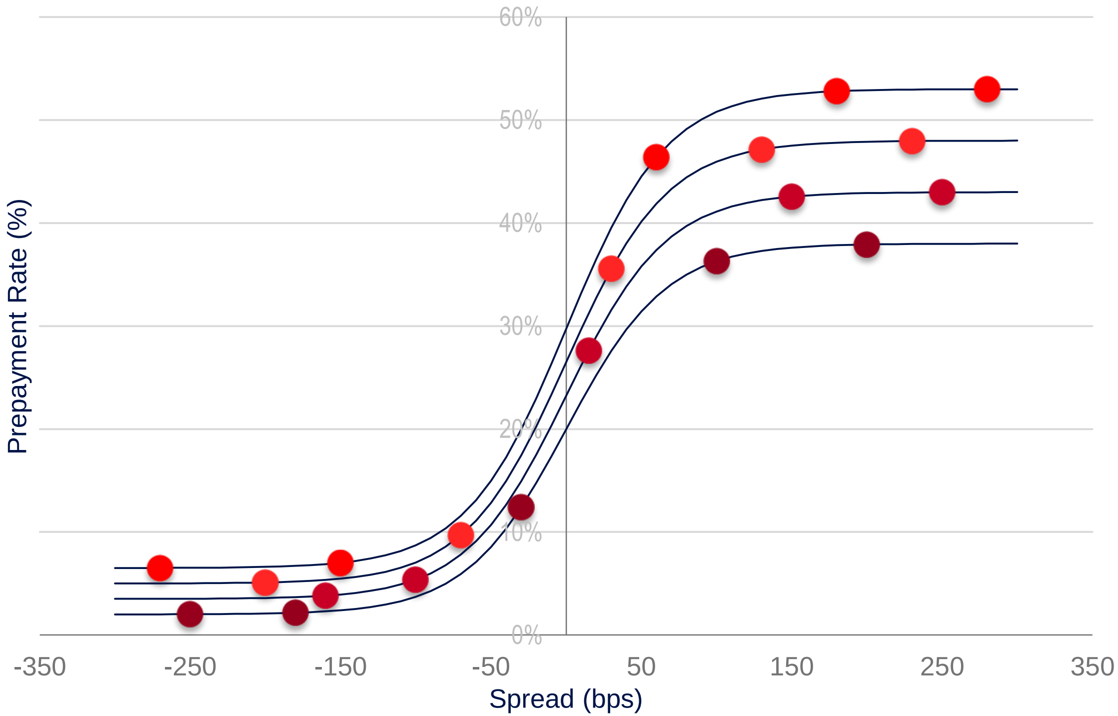
Granularity



Even if we assume homogenous behavior, prepayments may be significantly affected by **portfolio size and composition**

Aggregate redemption rates converge to the theoretical probability value in the granularity limit only.

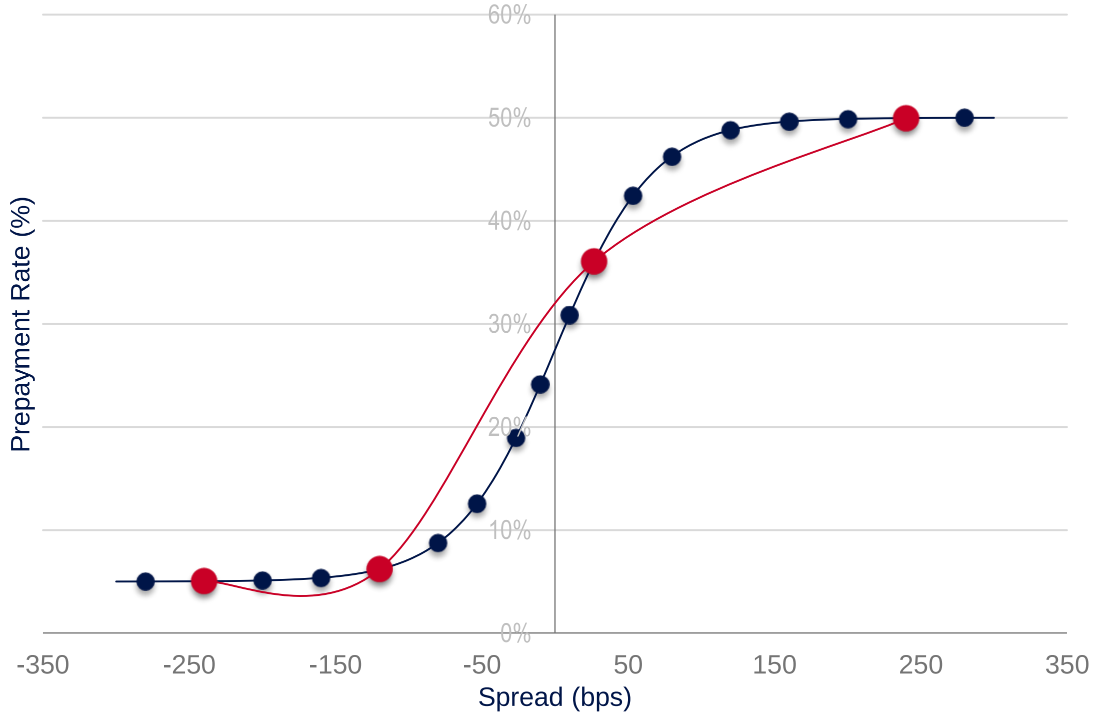
Exogenous factors



Prepayments are affected by **exogenous factors** besides financial reasons

- **Systemic factors** (such as GDP, employment rate, political uncertainty, etc.)
- **Individual factors** (such as relocation for mortgages or the contingent need of liquidity for deposits).

Estimation errors



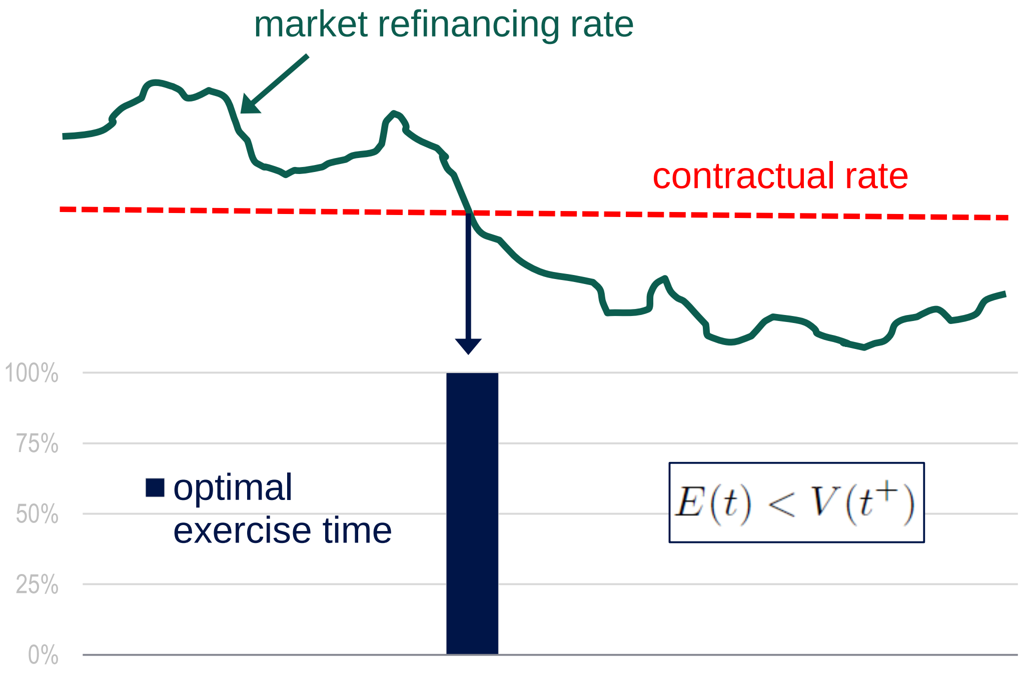
Extensive data-set is needed to achieve a robust calibration of a model

All models calibrated on historical basis face the same problem: does history repeat itself?

Estimation error should never be neglected or underestimated

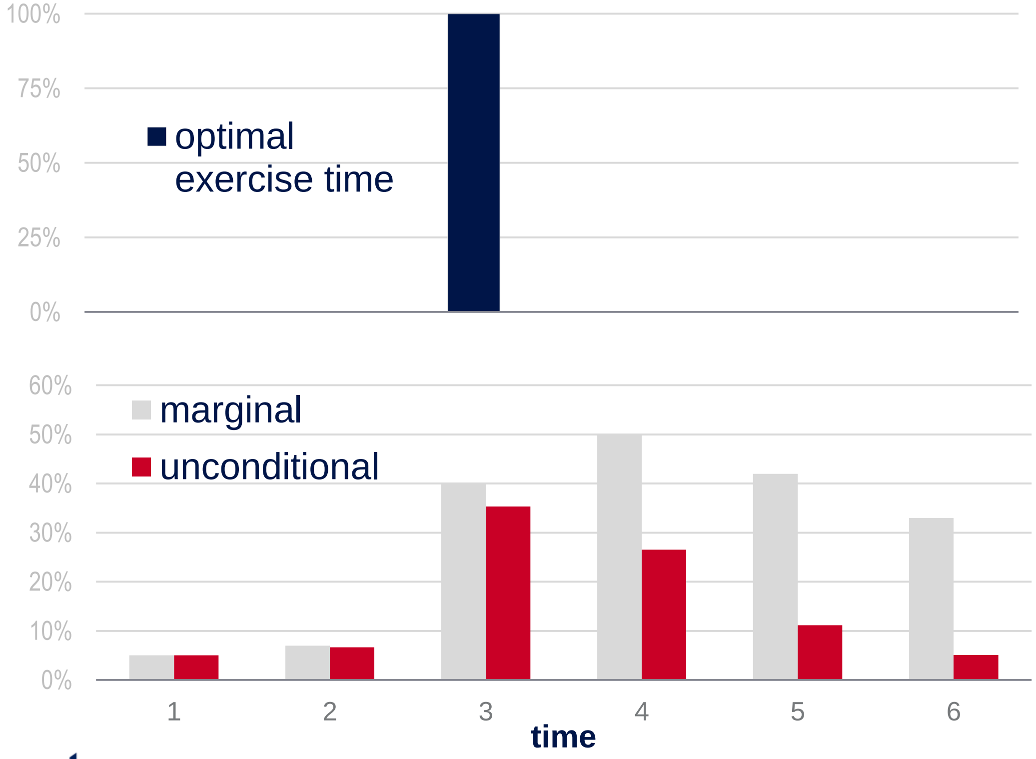
Compromise between sophistication and accuracy

Exercise times and cash flows “without” behavioral risk



- Rational option holders follow an exercise strategy maximizing their return
- For a mortgage with no prepayment penalty, optimal exercise time τ^* occurs when the exercise value (i.e. the remaining balance) is less than the continuation value
- Conditionally upon a market scenario and given a model for market factors, the **optimal exercise time is univocally determined**

Exercise times and cash flows “with” behavioral risk



- In the presence of behavioral risk the exercise time is random even when the market scenario is specified
- Only option exercise probabilities can be estimated
- We define behavioral risk by identifying the **additional cash flow variability** that it generates

Definition of behavioral risk

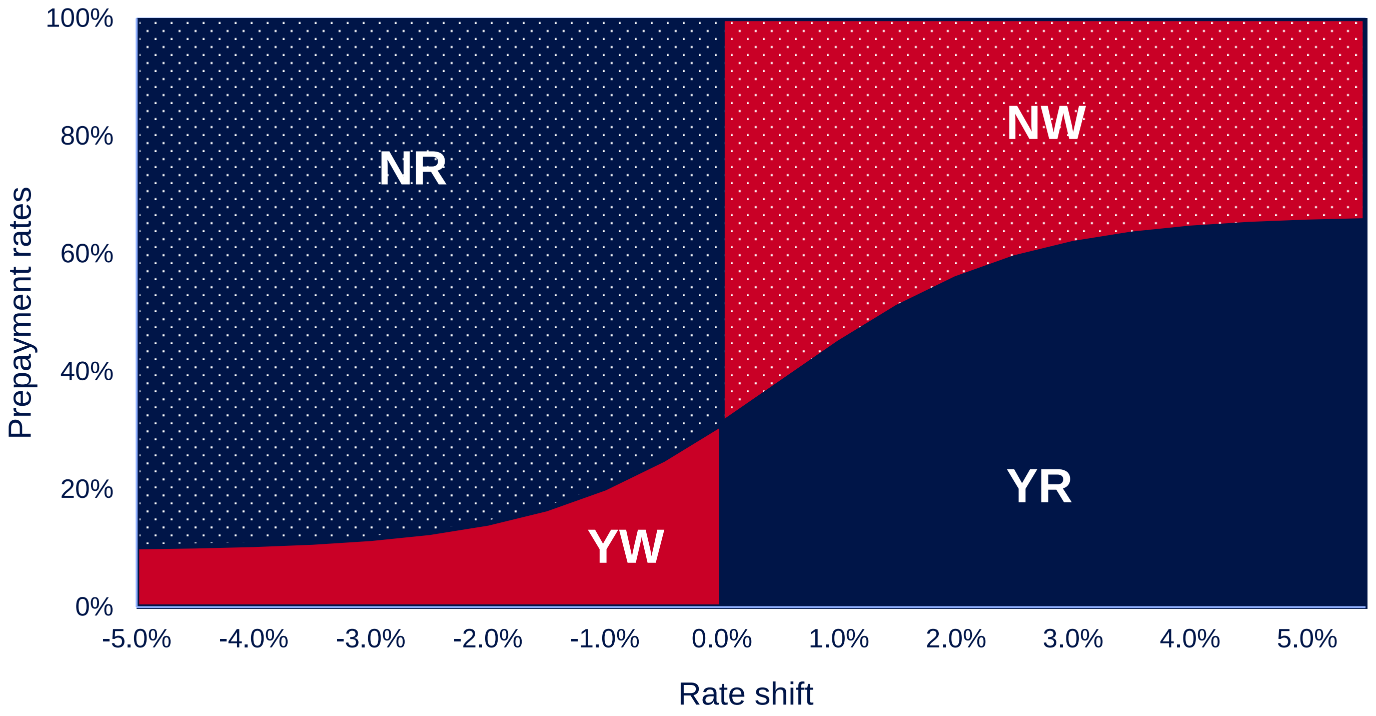
Therefore, we adopt the following definition:

Behavioral risk is the additional source of uncertainty in the future cash flows of a contract, when the option holder does not follow an optimal exercise strategy as seen from the point of view of the option seller.

In this way:

Behavioral risk is distinct from prepayment risk (or, more in general, option risk).

Rationality map



Sound models should account for deviation from fully rational behavior

Exercise decision:

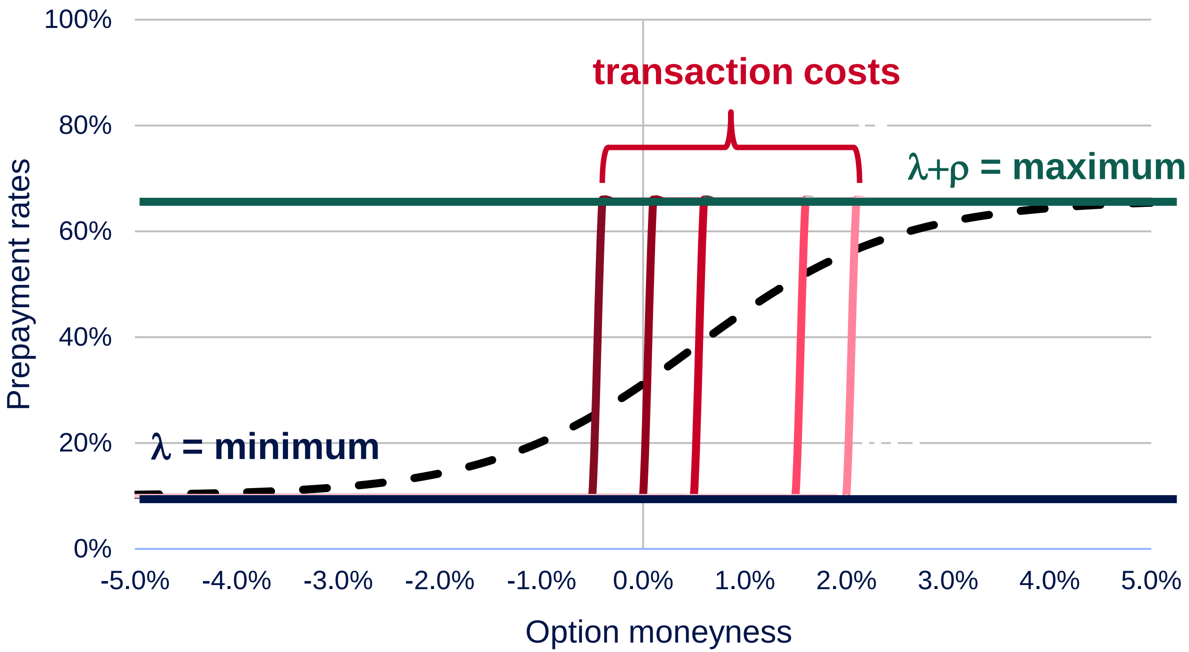
- Y (yes)
- N (no)

Financial convenience:

- R (right)
- W (wrong)

Behavioral models: option-based models

Option-based models start from the pure rational case and introduce frictions and transaction costs



- *Rational case for a callable contract*

$$Q_t = \begin{cases} 1 & E_t < V_t \\ 0 & \text{otherwise} \end{cases}$$

- *Stanton's model*

$$Q_t = \begin{cases} e^{-\lambda+\rho} & E_t + C_t < V_t \\ e^{-\lambda} & \text{otherwise} \end{cases}$$

Behavioral models: intensity models

Intensity-based models specify a hazard process for option exercise

$$h(t) = \underbrace{p_0(t)}_{\text{baseline}} \cdot \underbrace{e^{\sum_i b_i \cdot X_i(t)}}_{\text{scaling}}$$

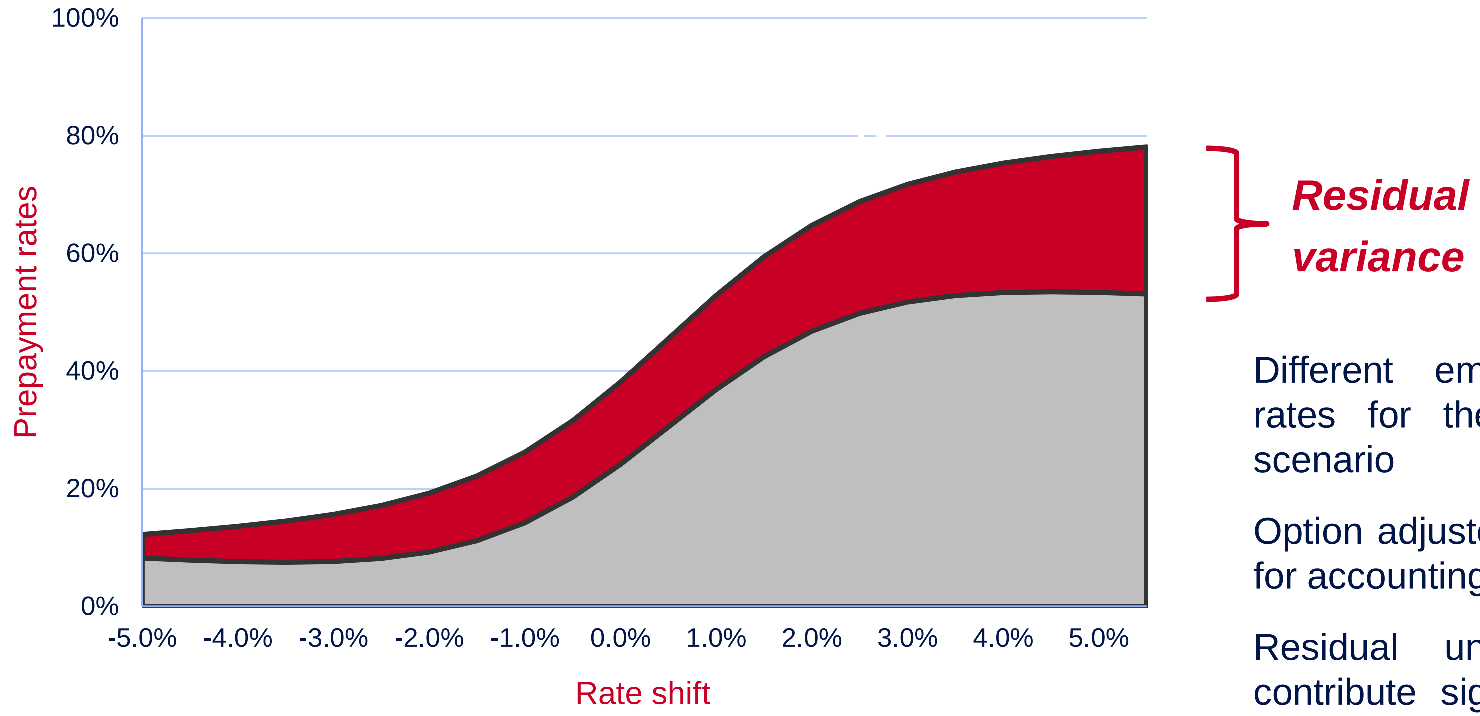
- *Schwarz/Torous model*

$$p_0(t,\gamma,\lambda) = \frac{\gamma\lambda(\gamma t)^{\lambda-1}}{1+(\gamma t)^{\lambda}},$$

$$\begin{aligned} x_1(t) &:= c - l(t-s) \\ x_2(t) &:= x_1(t)^3 \\ x_3(t) &:= \ln\frac{PF(t)}{A(t)} \end{aligned}$$

$$x_4(t) := \begin{cases} 1 & \text{if} \quad t = \text{May-August} \\ 0 & \text{if} \quad t = \text{September-April} \end{cases}$$

Modeling residual uncertainty

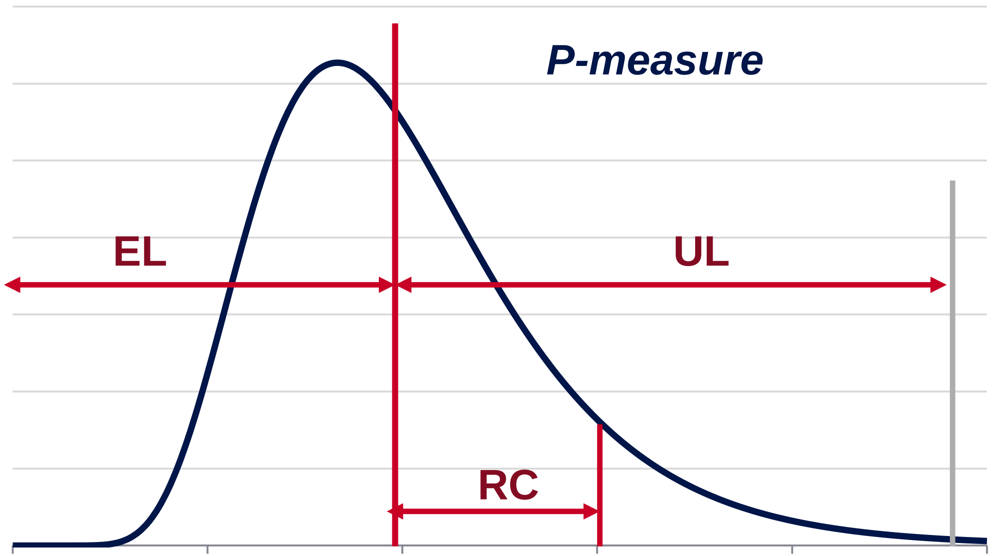


Different empirical exercise rates for the same market scenario

Option adjusted spread (OAS) for accounting for residual risk

Residual uncertainty might contribute significantly to the whole cash flow variance

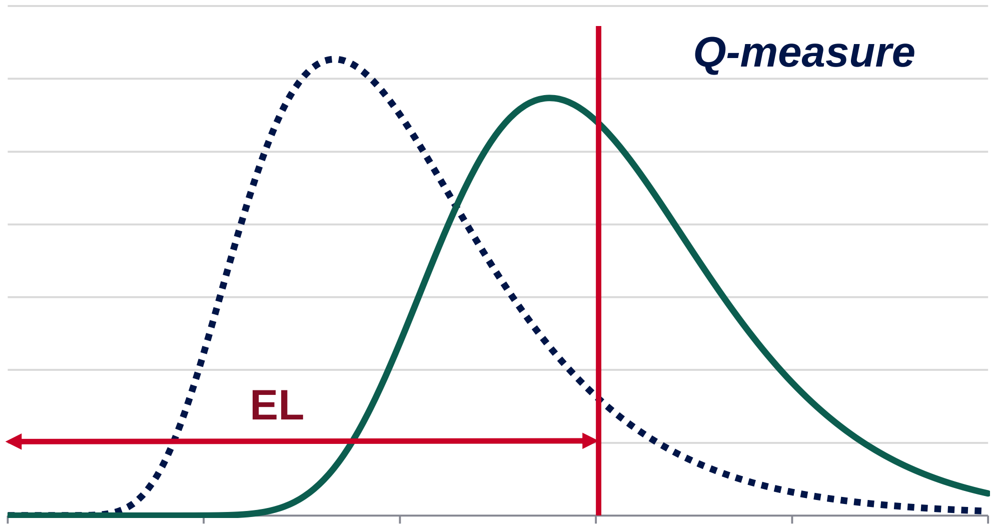
Risk-adjusted pricing through risk charge



$$V = \mathbb{E}_X^P[\Psi] - \mathrm{RC}$$

- The traditional risk-adjusted pricing approach consists in simulating the distribution of portfolio return under real-world probabilities
- Risk premium is the **cost of remunerating risky capital** needed to cover unexpected losses
- It depends on a target confidence level and hurdle rate for shareholders

Risk-neutral pricing through replication



$$V = \mathbb{E}_X^Q [\Psi]$$

The price of an instrument equals the cost of a self-financing hedging strategy

Risk premium is implied by market quotes and prices computed by simply taking expectations under risk-neutral probabilities

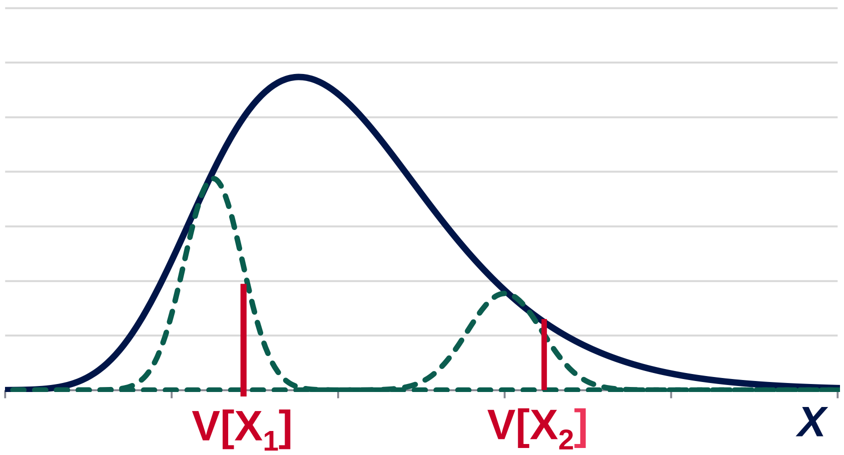
Hedging is often unfeasible since instruments in the replication portfolio are not traded or liquid

Risk can only be diversified in a large and granular portfolio

Behavioral risk premium

Since behavioral risk originates from a combination of market X and exogenous factors Z, we adopt a **mixed approach:**

- Risk neutral dynamics for market factors are calibrated from market quotes
- Exogenous factor dynamics are calibrated on historical basis



The price V(t) of a generic payoff Ψ is given by

expected & conditional unexpected cash flows

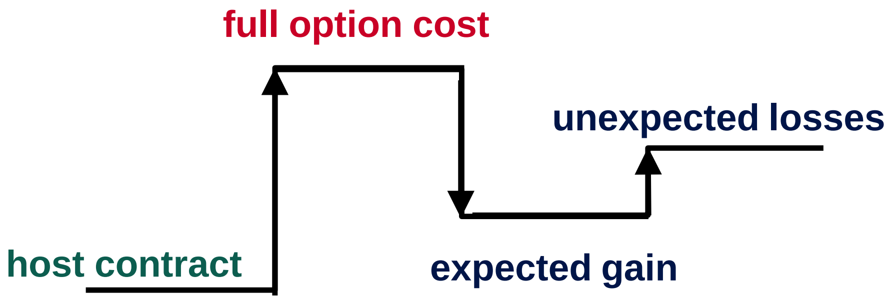
$$V(t) = V_E(t) - V_U(t) = \mathbb{E}_X^Q\left[\mathbb{E}_{Z,\tau}^P\left[\Psi|\vec{X}\right] - k \cdot \Phi_{Z,\tau}^P\left[\Psi|\vec{X}\right]\right]$$

Behavioral risk adjustments (βVA)

We can define behavioral-value adjustment (βVA) as

$$V(t) = \overbrace{V_H(t)}^{\text{host contract}} - \overbrace{\mathrm{OVA}(t)}^{\text{option adj.}} + \overbrace{\beta\mathrm{VA}(t)}^{\text{behavioral adj.}}$$

$$\underbrace{V_H(t) - \mathrm{OVA}(t)}_{\text{rational price } V_{sup}}$$



Behavioral-value adjustments can be split into two components having opposite sign

$$\beta\mathrm{VA}(t) = \beta\mathrm{VA}_E(t) - \beta\mathrm{VA}_U(t) \qquad \begin{cases} \beta\mathrm{VA}_E(t) = V_E(t) - V_{\mathrm{sup}}(t) \\ \beta\mathrm{VA}_U(t) = V_U(t) \end{cases}$$

Comment: XVA galaxy

Adjustments	
AVA	additional
BVA	bilateral
CVA	credit
DVA	debt
EVA	
FVA	funding
GVA	

Adjustments	
HVA	
IVA	
JVA	
KVA	capital
LVA	liquidity
MVA	margin
NVA	non-linearity

Adjustments	
OVA	option
PVA	(prudent)
QVA	
RVA	(rating)
SVA	
TVA	total or tax
UVA	unilateral

Adjustments	
VVA	
WVA	
YVA	
XVA	generic
ZVA	
ColVA	collateral

Behavioral risk adjustments (βVA) open the door to Greek alphabet

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Comment: hybrids model in hybrid markets

$$V = V_{RF} - \mathrm{CVA} - \mathrm{DVA} - \mathrm{LVA} - \mathrm{FVA} - \mathrm{KVA} - \mathrm{NVA}$$

derivative price

After the financial crisis, we have discovered that even plain vanilla instruments are **hybrid products,** subject to a combination of several risk sources

Adjustments are additive corrections that might underestimate **non-linearity** effects between different risk factors

Behavioral value adjustment can be interpreted as:

- *Conditional premium*
- *Hedging error*
- *Capital remuneration up to contract maturity*

Parallel with credit risk modelling

- **Credit Risk**
  - *Defaultable asset*
  - *Defaultable counterpart*
  - *Default event*
  - *Debtor creditworthiness*

- **Behavioral Risk**
  - *Early-exercise option*
  - *Option holder*
  - *Option exercise*
  - *Investor behavior*

Microstructural approach

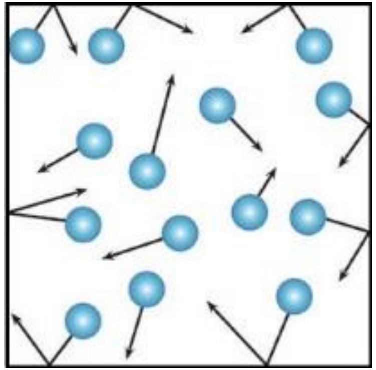
Microstructural approach. For each *p*-th contract and *i*-th investor, the marginal probability of option exercise is a function of a set of market and exogenous factors (X, Z)

$$Q^{ip}(t) = R\left(\vec{X}(t), \vec{Z}(t), \vec{\theta}\right)$$

Market factors affect both contractual payments and exercise decisions

Individual exogenous factors are specified for all investors (each one having a systemic and an idiosyncratic component), like in the Vasicek model for credit risk

$$Z^i(t) = \rho \cdot \xi^0(t) + \sqrt{1-\rho^2} \cdot \ \xi^i(t)$$

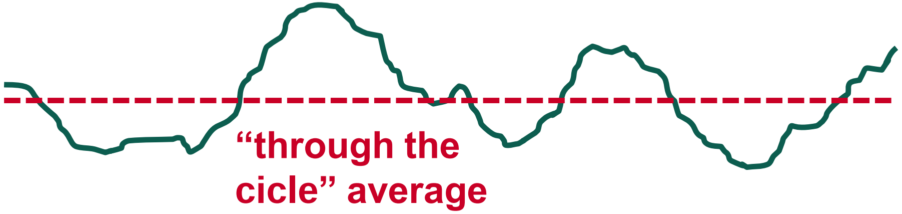


$$PV = nRT$$

Mathematical framework

- **Long term averaging**. We assume that the effect of exogenous factors tends to cancel out over a long period of time

$$\mathbb{E}^P[Z_\infty] = 0, \qquad \mathbb{V}^P[Z_\infty] = 1$$



- **Conditional independence**. Subject to the realization of a macro-scenario (X, Z), prepayment decisions are taken independently by different investors for each contract type.

- **Coherent risk measure** (such as Expected Shortfall), linked to the capital absorption needed to compensate for unexpected losses through the entire life of the contract. If the distribution is not excessively skewed we can choose

$$\Phi^P(\Psi|X) = \chi_q \cdot \sqrt{\mathbb{V}[\Psi|X]}$$

General payoffs

Single contract discounted payoff, depending on exercise time τ

cash flows **no exercise** **&** **early exercise**

$$\Psi = \underbrace{\sum_{k=1}^{T} D_k \cdot C_k \cdot \mathbb{I}(\tau > t_k)}_{\text{no exercise}} + \underbrace{\sum_{k=1}^{T} D_k \cdot E_k \cdot \mathbb{I}(\tau = t_k)}_{\text{early exercise}}$$

General formula for discounted portfolio payoff of instruments with embedded options

number of investors **number of contracts**

$$\Psi = \sum_{i=1}^{N} \sum_{p=1}^{M} N^{ip} \cdot \left(\sum_{k=0}^{T} D_k \cdot M_k^p \cdot \mathbb{I}(\tau^{ip} > t_k) \right)$$

$$N^p = \sum_{i=1}^{N} N^{ip}$$

number of *p*-th contracts held by the *i*-th investor

Portfolio pricing

General formula for portfolio pricing $$V(0) = \mathbb{E}_X^Q \left[\Pi_0(X) - k \cdot \chi_q \cdot \sqrt{\Pi_1(X) + \Pi_2(X)} \right]$$

$$\Pi_0(X) = \mathbb{E}_Z^P \left[\mathbb{E}_\tau^P \left[\Psi \middle| X, Z \right] \middle| X \right] = \sum_{i=1}^{N} \sum_{p=1}^{M} \sum_{k=0}^{T} L_k^{ip} \cdot \mathbb{E}_Z^P \left[S_k^{ip} \middle| X \right]$$

revised cash flow expectation

$$\Pi_1(X) = \mathbb{E}_Z^P \left[\mathbb{V}_\tau^P \left[\Psi \middle| X, Z \right] \middle| X \right] = \sum_{i=1}^{N} \sum_{p=1}^{M} \sum_{k,h=0}^{T} L_k^{ip} L_h^{ip} \cdot \mathbb{E}_Z^P \left[S_{\max(k,h)}^{ip} - S_k^{ip} S_h^{ip} \middle| X \right]$$

granularity effect

$$\Pi_2(X) = \mathbb{V}_Z^P \left[\mathbb{E}_\tau^P \left[\Psi \middle| X, Z \right] \middle| X \right] = \sum_{i,j=1}^{N} \sum_{p,q=1}^{M} \sum_{k,h=0}^{T} L_k^{ip} L_h^{jq} \cdot \mathbb{V}_Z^P \left[S_k^{ip}, S_h^{jq} \middle| X \right]$$

variance induced by exogenous factors

with $L_k^{ip} = N^{ip} \cdot D_k \cdot M_k^p$

Granularity limit

well-diversified portfolio

$$\begin{cases} N >> 1 \\ N^{ip} \approx \frac{N^p}{N} \\ \mathbb{V}_Z^P \left[S_k^{ip}, S_h^{jq} \middle| X \right] = 0 \quad \forall i \neq j \end{cases}$$

large number of counterparts

equally-sized contracts

purely idiosyncratic exogenous factors

behavioral risk is fully diversified and no βVA_U is needed

$$V(0) \approx \mathbb{E}_X^Q \left[\Pi_0(X) \right]$$

granular portfolio

$$\begin{cases} N >> 1 \\ N^{ip} \approx \frac{N^p}{N} \end{cases}$$

$$V(0) \approx \mathbb{E}_X^Q \left[\Pi_0(X) - k \cdot \chi_q \cdot \sqrt{\Pi_2(X)} \right]$$

granularity indicator

$$H^p = \sum_{i=1}^{N} \left(\frac{N^{ip}}{N^p} \right)^2$$

Herfindahl-Hirschman Index

$H^p \to 0$ in the granularity limit

Basel perspective: IRRBB revised standards (2016)

According to principles released by the Basel Committee in 2004, interest rate risk in the banking book has to be managed under Pillar 2

In 2015 Basel Committee has proposed a new treatment of (IRRBB) by suggesting the introduction of a **standardized framework** (2015). The goal was to achieve:

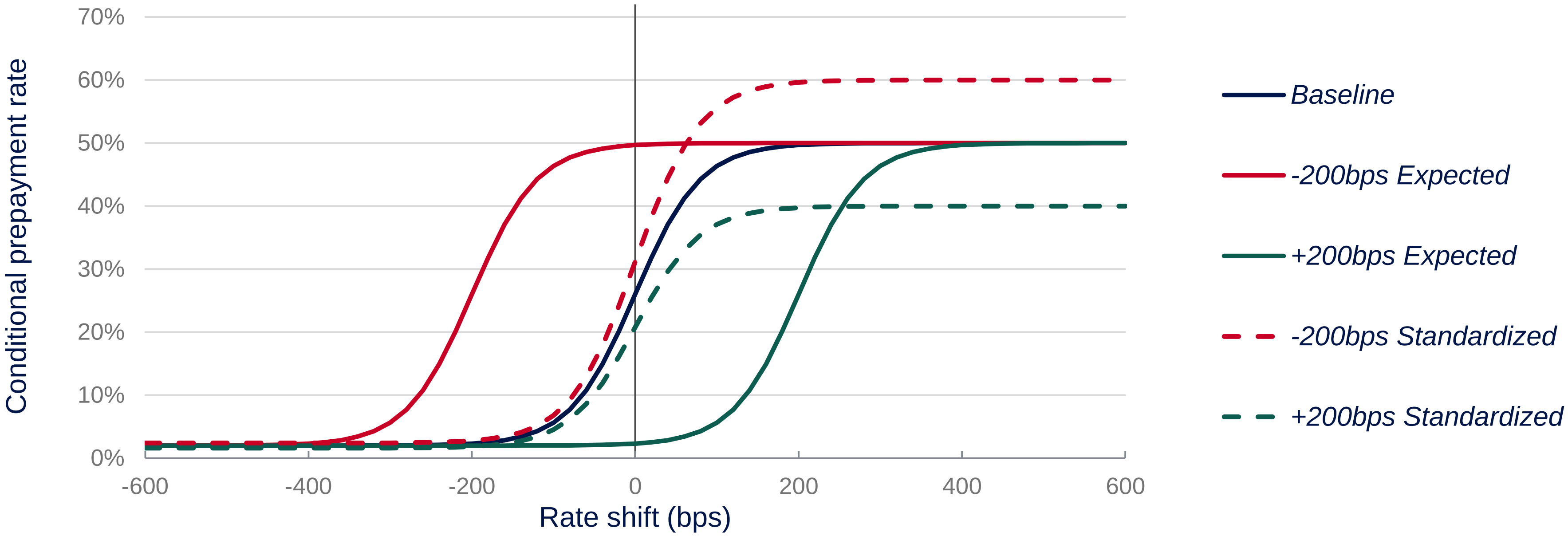
- *Standardization of the approach with improved comparability*
- *Reinforce capital requirements that might be underestimated by internal models*

IRR is measured on the basis of a set of stress scenarios on both economic value and earnings, but with a rather **unrealistic modeling of behavioral risk**

Following negative feedback from banking industry, Basel Committee has released new standards (2016), where the standardized approach is not mandatory and IRRBB can still be implemented **under revised Pillar 2**

Basel perspective: standardized approach for behavioral risk

- In the standardized approach baseline exercise rates (CPR) are assumed for prepayment options. Static CPR multipliers are introduced for each stress scenario. **No additional variance** is considered at all.



Basel perspective: βVA and KVA

In the pricing formula, hedging costs (V_{sup}) and traditional risk charge (βVA) remunerate market/prepayment and behavioral risk, separately. In reality, **both approaches may underestimate total risk**.

Regulations impose capital requirements K that imply an overlap between them, with a sort of double counting of risk premium. Can standardized approach be a benchmark, although not mandatory?

$$V(t) = V_H(t) - \mathrm{OVA}(t) + \beta\mathrm{VA}_E(t) - \beta\mathrm{VA}_U(t)$$

full behavioral risk valuation

$$V(t) = V_H(t) - \mathrm{OVA}(t) + \beta\mathrm{VA}_E(t) - \mathrm{KVA}(t)$$

economic capital

with $$\mathrm{KVA}(t) = k \cdot \mathbb{E}^Q \left[\int_t^T D(t,s) \cdot K(s) \cdot ds \right]$$

Effective behavioral value adjustment? $$\beta\mathrm{VA}(t) = \beta\mathrm{VA}_E(t) - \max\left(\beta\mathrm{VA}_U(t), \mathrm{KVA}(t)\right)$$

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Thank you for your attention!

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Appendix

A specific behavioral intensity hybrid model (BIX)

- We assume the following response function R

$$\ln\left[1 - Q_k^{ip}\right] = \underbrace{A^{ip}\left(t_k, \vec{X}_k\right)}_{\text{average responsiveness to market factors (fit)}} + \underbrace{\overbrace{B^{ip}\left(t_k, \vec{X}_k\right)}^{\text{conditional standard deviation}} \cdot Z_k^i}_{\text{error process (residuals)}}$$

- Lognormal conditional survival probabilities

$$S_k^{ip}(X, Z) = \prod_{h=1}^{k}\left[1 - Q_h^{ip}(X, Z)\right] = e^{W_k^{ip}(X,Z)}$$

- Exogenous factors are modeled by AR(1) process with parameters $\left\{\rho, \alpha, \beta, \xi_0^0, \xi_0^i = 0 \quad \forall i > 0\right\}$

$$Z_k^i = \rho \cdot \xi_k^0 + \sqrt{1 - \rho^2} \cdot \ \xi_k^i$$
$$\xi_k^i = \alpha \cdot \xi_{k-1}^i + \beta \cdot \varepsilon_k^i$$

$$\begin{cases} \mathbb{E}_Z^P\left[\xi_\infty | X\right] = 0 \ \Rightarrow \ |\alpha| < 1 \\ \mathbb{V}_Z^P\left[\xi_\infty | X\right] = 1 \ \Rightarrow \ \beta = \sqrt{1 - \alpha^2} \end{cases}$$

Pricing of a homogeneous portfolio I

Homogeneous portfolio pricing

$$V(0) = \mathbb{E}_X^Q \left[\Pi_0(X) - k \cdot \chi_q \cdot \sqrt{\Pi_1(X) + \Pi_2(X)} \right]$$

$$A_k^{ip} = A_k^p, \;\; B_k^{ip} = B_k^p$$

- The first two terms $\Pi_0(X)$ and $\Pi_1(X)$ are linear with respect to the number of contracts

$$\Pi_0(X) = \sum_{p=1}^{M} \left[L_0^p(X) + \sum_{k=1}^{T} L_k^p(X) \cdot E_k^p(X) \right]$$

$$\Pi_1(X) = \sum_{p=1}^{M} \left[H^p \cdot \sum_{k=0}^{T} L_k^p(X) \cdot I_k^p(X) \cdot E_k^p(X) \right]$$

with

$$L_k^p(X) = N^p \cdot D_k \cdot M_k^p \qquad E_k^p(X) = e^{\mu_k^p(X) + \frac{1}{2}\sigma_k^{p\,2}(X)}$$

$$I_k^p(X) = L_k^p(X) \cdot \left(1 - e^{\mu_k^p(X) + \frac{3}{2}\sigma_k^{p\,2}(X)}\right) + 2 \cdot \sum_{h=0}^{k-1} L_h^p(X) \cdot \left(1 - e^{\mu_h^p(X) + \frac{3}{2}\sigma_h^{p\,2}(X)}\right)$$

Pricing of a homogeneous portfolio II

- Since the last term Π_2 corresponds to the variance of a weighted sum of lognormal variables, we rely on Gentle's approximation which was originally developed for the pricing of Asian options within BS framework

$$\Pi_2(X) = \mathbb{V}_Z^P\left[\sum_{p=1}^{M}\sum_{k=1}^{T}\sum_{i=1}^{N} L_k^{ip}(X)\cdot S_k^{ip}(X,Z)\middle| X\right]$$
$$\approx \mathbb{V}_Z^P\left[\prod_{p=1}^{M}\prod_{k=1}^{T}\prod_{i=1}^{N} e^{L_k^{ip}(X)\cdot W_k^{ip}(X,Z)}\middle| X\right] = \mathbb{V}_Z^P\left[e^{\Omega(X,Z)}\middle| X\right]$$

$$\Pi_2(X) \approx e^{2\cdot M_\Omega(X)+\Sigma_\Omega^2(X)}\cdot\left(e^{\Sigma_\Omega^2(X)}-1\right)$$

In practice, one can simulate several market scenarios X, compute conditional values Π_0(X), Π_1(X), Π_2(X), and then apply

$$V(0) = \mathbb{E}_X^Q\left[\Pi_0(X) - k\cdot\chi_q\cdot\sqrt{\Pi_1(X)+\Pi_2(X)}\right]$$