Portfolio construction: "factors"

Policy	Signal (function of current information)		Characteristics ("Expected return" from the signal)		Portfolio (vector of holdings)	
$h(\cdot):$	$\boldsymbol{s}_t \equiv \begin{pmatrix} s_{1,t} \\ \cdot \\ s_{\bar{n},t} \end{pmatrix} \in \boldsymbol{i}_t$	$\rightarrow$	$\boldsymbol{\beta}_t^{signal} \equiv \begin{pmatrix} \beta_{1,t}^{signal} \\ \cdot \\ \beta_{\bar{n},t}^{signal} \end{pmatrix}$	$\rightarrow$	$\boldsymbol{h}_t^{signal} \equiv \begin{pmatrix} h_{1,t}^{signal} \\ \cdot \\ h_{\bar{n},t}^{signal} \end{pmatrix}$	(9b.4)

Characteristic portfolio

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- **Characteristic signal portfolio**

$$\boldsymbol{h}_t^{signal} \equiv \underset{\boldsymbol{h}'\boldsymbol{\beta}_t^{signal}=1}{\operatorname{argmin}} \ \mathbb{V}\{\Pi_{\boldsymbol{h},t\rightarrow t+1}|\boldsymbol{s}_t\} = \underset{\boldsymbol{h}'\boldsymbol{\beta}_t^{signal}=1}{\operatorname{argmin}} \ \boldsymbol{h}'\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t}\boldsymbol{h} \qquad (9b.35)$$

↓ signal characteristics (9b.18)

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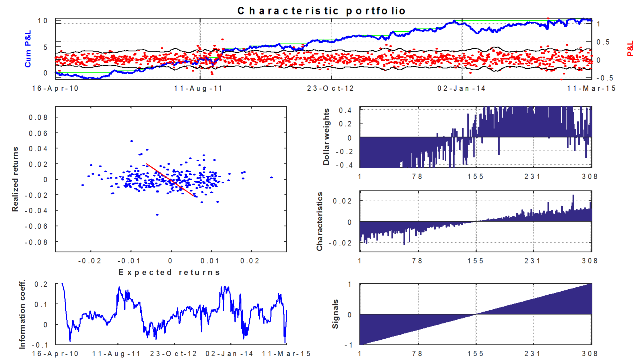
⇓

Analytical solution

$$\boldsymbol{h}_t^{signal} = \frac{(\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t})^{-1}\boldsymbol{\beta}_t^{signal}}{\boldsymbol{\beta}_t^{signal\prime}(\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t})^{-1}\boldsymbol{\beta}_t^{signal}} \qquad (9b.36)$$

Signal characteristic portfolio stock market (E)

Characteristic portfolio of reversal strategy



- **Risk drivers**: log-values of 392 stocks of the S&P500 index
- **Signal** : negative momentum (reversal)

Flexible portfolio construction

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$$\boldsymbol{h}_t^{signal} = \underset{\boldsymbol{h}'\boldsymbol{\beta}_t^{char}=1}{\operatorname{argmin}} \; \boldsymbol{h}'\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t}\boldsymbol{h} \tag{9b.35}$$

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$$\Updownarrow$$

Dual Problem:

$$\boldsymbol{h}_t^{signal} \equiv \operatorname{argmax}_{\boldsymbol{h}\in\mathcal{C}_t}\{\boldsymbol{h}'\boldsymbol{\beta}_t^{signal}\} \tag{9b.43}$$

$$\mathcal{C}_t \equiv \{\boldsymbol{h}'\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t}\boldsymbol{h} \leq \frac{1}{\boldsymbol{\beta}_t^{char\prime}(\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t})^{-1}\boldsymbol{\beta}_t^{char}}\} \tag{9b.44}$$

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Policy	Signal (function of current information)		Characteristics ("Expected return" from the signal)		Portfolio (vector of holdings)	
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Dual problem (9b.43)-(9b.44) allows to consider more flexible constraints

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$$\{\boldsymbol{h}'\boldsymbol{\sigma}_{\boldsymbol{\Pi};t}^2\boldsymbol{h} \leq \bar{\sigma}_t^2\} \subseteq \mathcal{C}_t \quad (9b.45)$$

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$$\{\underline{v}_t \leq \boldsymbol{h}'\boldsymbol{v}_t \leq \bar{v}_t\} \subseteq \mathcal{C}_t \quad (9b.46)$$

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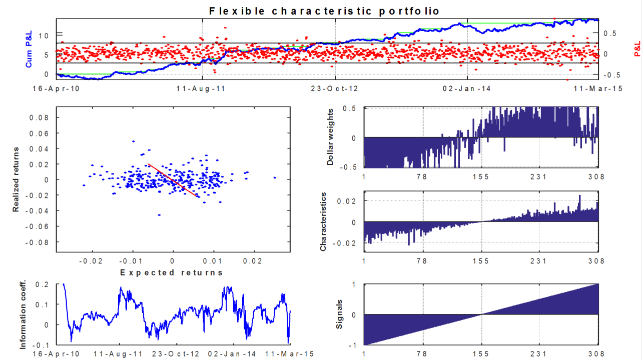
- **self-financing** or **full-investment** constraint

$$\{\boldsymbol{h}'\boldsymbol{v}_t = v_{0,t}\} \subseteq \mathcal{C}_t \tag{9b.47}$$

↓

$v_{0,t} = 0 \Rightarrow$ dollar-neutral constraint (9b.48)

Flexible characteristic portfolio of reversal strategy



- **Risk drivers**: log-values of 392 stocks of the S&P500 index
- **Signal** : negative momentum (reversal)
- **Constraints**: bound of variance, market neutrality, with stock index, zero investment long/short

Flexible portfolio construction

multiple $\bar{d}$-dimensional signals

$$\boxed{\boldsymbol{\beta}_t^{signal}} \equiv (\underset{\bar{n}\times 1}{\boldsymbol{\beta}_t^{signal_1}} | \cdots | \underset{\bar{n}\times 1}{\boldsymbol{\beta}_t^{signal_{\bar{k}}}}) \quad (9b.49)$$

number of instruments in the strategy ← $\bar{n} \times \bar{k}$ → number of signal types

Flexible portfolio construction

multiple $\bar{d}$-dimensional signals $\boxed{\boldsymbol{\beta}_t^{signal}} \equiv (\boldsymbol{\beta}_t^{signal_1} | \cdots | \boldsymbol{\beta}_t^{signal_{\bar{k}}})$ (9b.49)

$\bar{n} \times 1$ $\bar{n} \times 1$

number of instruments in the strategy ← $\bar{n} \times \bar{k}$ → number of signal types

$$\Downarrow$$

$$\boldsymbol{h}_t^{signal_k} \equiv \underset{\boldsymbol{h} \in \mathcal{C}_t}{\text{argmax}} \{\boldsymbol{h}' \boldsymbol{\beta}_t^{signal_k}\} \quad (9b.50)$$

$$\left\{ \begin{array}{ll} \boldsymbol{h}^{k\prime} \boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t} \boldsymbol{h}^k \leq \dfrac{1}{\boldsymbol{\beta}_t^{signal_k \prime} (\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t})^{-1} \boldsymbol{\beta}_t^{signal_k}} & (9b.51) \\ \boldsymbol{h}^{k\prime} \boldsymbol{\beta}_t^{signal_j} = 0, \quad \text{for } j \neq k & (9b.52) \end{array} \right\} \subseteq \mathcal{C}_t$$

Flexible portfolio construction

multiple $\bar{d}$-dimensional signals

$$\underbrace{\boldsymbol{\beta}_t^{signal}}_{\bar{n}\times\bar{k}} \equiv (\underbrace{\boldsymbol{\beta}_t^{signal_1}}_{\bar{n}\times 1}|\cdots|\underbrace{\boldsymbol{\beta}_t^{signal_{\bar{k}}}}_{\bar{n}\times 1}) \quad (9b.49)$$

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⇓

multi-signal char. portfolios $$(\underbrace{\boldsymbol{h}_t^{signal_1}}_{\bar{n}\times 1}|\cdots|\underbrace{\boldsymbol{h}_t^{signal_{\bar{k}}}}_{\bar{n}\times 1}) = \boldsymbol{\beta}_t^{signal\dagger} \quad (9b.53)$$

↓

optimal pseudo-inverse (9b.54)

Flexible portfolio construction

Relevant constraints

- **Uncorrelation** constraint

$$\{\boldsymbol{h}^{k\prime}\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t}\underbrace{\tilde{\boldsymbol{h}}_t}_{\text{exogeneous portfolios}} = 0\} \subseteq \mathcal{C}_t \tag{9b.55}$$

Flexible portfolio construction

Relevant constraints

- **Uncorrelation** constraint

$$\{\boldsymbol{h}^{k\prime}\boldsymbol{\sigma}^2_{\boldsymbol{\Pi};t}\tilde{\boldsymbol{h}}_t = 0\} \subseteq \mathcal{C}_t \quad (9b.55)$$

↓ exogeneous portfolios

- **market impact/trans. costs** penalty

$$\boldsymbol{h}_t^{signal_k} \equiv \underset{\boldsymbol{h}^k \in \mathcal{C}_t}{\operatorname{argmax}}\{\boldsymbol{h}^{k\prime}\boldsymbol{\beta}_t^{signal_k} - \lambda c(\boldsymbol{h}^k, \boldsymbol{h}^k_{t-1})\} \quad (9b.56)$$

↓

$$c(\boldsymbol{h}, \boldsymbol{k}) \equiv (\boldsymbol{h} - \boldsymbol{k})'\boldsymbol{q}^2(\boldsymbol{h} - \boldsymbol{k}) \quad (9b.57)$$